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NOMOGRAMS FOR THE DESIGN OF SINGLE SAMPLE RELIABILITY ACCEPTANC--ETC(U)

JAN 77 A L GOEL, A M JOGLEKAR F30602-71-C-0312

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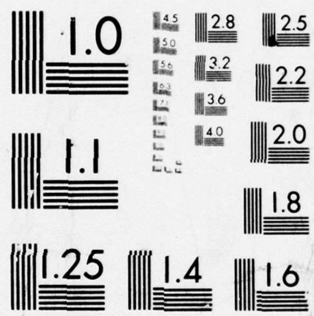
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Final Technical Report
January 1977

NOMOGRAMS FOR THE DESIGN OF SINGLE SAMPLE RELIABILITY ACCEPTANCE PLANS FOR EXPONENTIAL DISTRIBUTION

Syracuse University

ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents nomograms for the design and evaluation of single sample reliability acceptance tests for exponential distribution. The nomograms permit an exploration of the design region and facilitate the examination of alternative solutions and their implications. The nomograms constitute a useful tool for the practicing engineer to weigh the trade-offs between design and test criteria. Several examples are presented to illustrate the use of the nomograms toward plan design, exploration of the design region, → next page

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examination of the effects of changes in design criteria on the designed plans, design with engineering constraints and design with partial information.

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EVALUATION

This study was one of many responsive to Technical Planning Objective #13 (Reliability) performed by Syracuse University under Contract F30602-71-C-0312. The objective of this study was to provide a graphical means for designing reliability tests. This objective was achieved and the report provides the graphs and the necessary instructions for use. Besides the dissemination of the report to potential users, follow-on activity is currently in progress to develop more efficient statistical test methods, for ultimate incorporation into MIL-STD-781, Reliability Tests: Exponential Distribution.

Anthony Coppola

ANTHONY COPPOLA
Project Engineer

TABLE OF CONTENTS

1. INTRODUCTION

This paper presents nomograms for the design of single sample reliability acceptance tests for exponential failure distribution given by:

$$f(t|\theta) = \frac{1}{\theta}e^{-t/\theta} \quad ; \quad t \geq 0, \ \theta > 0 \qquad (1)$$

where t denotes the time to failue and θ is the mean time between failures (MTBF). The nomograms are applicable when accept/reject decisions are to be made regarding a sequence of systems or a sequence of lots. It is assumed that the decisisons are based upon data subject to random fluctuations, that a definite measure of loss is associated with each of the decisions where they are inappropriately taken and that each lot or system is sentenced on its own merit without regard to the previous decisions.

Single sample tests for lots or systems may be truncated or censored. A lot consists of a large number of components from which only a small sample is usually life tested. Each repairable system is presumed to be individually tested. In a truncated test, a system (or a random sample of n items from the lot) is life tested for duration T (or t_o). If the observed number of failures r is less than or equal to the acceptance number r*, the system (lot) is accepted. Otherwise, it is rejected. In censored testing, the life test is continued till exactly r' failures occur. If the total test time is greater than T', the decision is to accept. Otherwise, the decision is to reject. For the case of a lot, testing may be done with replacement, where the failed items are immediately replaced or without replacement where the failed items are not replaced. For a single system testing without replacement is not possible. The various plans described above are interrelated such that if one plan is known, others can be readily obtained. Therefore, in the following, we consider the design of a single sample truncated plan for a system.

The plans are designed to distinguish between two values of θ, namely, the minimum acceptable value θ_1 and the specified value θ_0. The risks associated with the reject/accept decisions when they are inappropriately taken are called the producer's risk and the consumer's risk and are respectively defined as

$$P(R|\theta = \theta_0) = \alpha \tag{2}$$

and

$$P(A|\theta = \theta_1) = \beta \tag{3}$$

where R denotes rejection and A denotes acceptance. Typically α and β are assigned small numerical values.

Conventional Design: We now consider the conventional design of a single sample truncated plan for a system where the design parameters are the test time T and the acceptance number r*. Since the time to failure is exponential, the observed number of failures r in fixed time T has a Poisson distribution, i.e.

$$f(r|\theta) = \frac{e^{-T/\theta}\,(T/\theta)^r}{r!} \tag{4}$$

The producer's and consumer's risk can be written as

$$\sum_{r=0}^{r^*} \frac{e^{-T/\theta_0}(T/\theta_0)^r}{r!} = 1 - \alpha, \text{ and} \tag{5}$$

$$\sum_{r=0}^{r^*} \frac{e^{-T/\theta_1} \; (T/\theta_1)^r}{r!} = \beta \tag{6}$$

Given (θ_1, θ_0,α,β), equations (5) and (6) can be simultaneously solved to obtain T and r*.

Individual specification of θ_1 and θ_0 is not necessary. Let the discrimination ration K= θ_0/θ_1 and let T*= T/θ_0. Then,

$$\sum_{r=0}^{r^*} \frac{e^{-T^*}(T^*)^r}{r!} = 1 - \alpha \tag{7}$$

and

$$\sum_{r=0}^{r^*} \frac{e^{-KT^*}(KT^*)^r}{r!} = \beta \tag{8}$$

Given (K,α,β) the above two equations can be solved to obtain T^* and r^*. Clearly, plans with identical θ_0/θ_1 have the same T^* and r^* values. Given the specified value θ_0, the actual test time is obtained as $T= \theta_0 T^*$.

Conventionally, the equations are solved numerically or by using tables of cumulative Poisson probabilities. For various numerical values of α, β and K, the solutions are tabulated in standards such as 781 B [2].

Purpose and Scope: The conventional approach merely provides a designed plan for specified (K,α,β). It does not easily lend itself to an examination of alternative solutions and the implications of K,α,β on T^* and r^*. The study reported here was motivated by a desire to provide a tool to the practicing engineer to help arrive at a meaningful test plan under various practical situations. We first present a graphical procedure which permits an exploration of the design region leading to the preparation of tables summarizing the region of interest. These alternatives serve as a guide to consider the trade-offs between α, β, K, T^* and r^*. The application of the graphical procedure to situations of practical interest is illustrated via several examples.

2. GRAPHICAL PROCEDURE

The graphical procedure consists of drawing contours of constant α and β in the (T^*-r^*) plane. The contours are obtained as follows. For given values of T^*, r^* and K, equations (7) and (8) can be used to obtain α and β. For a constant K, a grid of T^*, r^* values is constructed to cover appropriate ranges of T^* and r^*. The values of α and β are evaluated at each of the grid points and contours of constant α and constant β are drawn by parabolic interpolation. For specified K, α,β these contours can be used to design and evaluate the test plans.

Eight values of discrimination ratio are considered to cover the useful range $K = 1.5\ (0.5)5$. The corresponding nomograms are given in Figures 1 to 8. For each nomogram, the range of T^* and r^* values is appropriately chosen to include the region of interest. The contours are plotted for several practically useful values of α and β. It is easily observed from equations (7) and (8) that a change in K changes β but has no influence on α. Hence the contours of the producer's risk are identical for all eight nomograms, except for the changes in scale and range of values.

We now present several examples to show the important features of the graphical procedure. Specifically, the examples illustrate (a) design of single sample plan, (b) exploration of the design region, (c) sensitivity of the T^* and r^* values to changes in α,β and K, (d) design with engineering constraints, and (e) design with partial information.

3. ILLUSTRATIVE EXAMPLES

3.1 Example 1: Design

A single sample fixed time acceptance plan is to be obtained for $K = 3$, $\alpha \leq 0.05$ and $\beta \leq 0.05$. For this discrimination ratio, we use Figure 4 and consider the contours for $\alpha = 0.05$ and $\beta = 0.05$. The point of intersection of these contours is between the $r^* = 8$ and $r^* = 9$ lines slightly to the right of $T^* = 5$. The triangular area enclosed by $\alpha = .05$ and $\beta = .05$ lines represents the solution region within which the requirements $\alpha \leq 0.05$ and $\beta \leq 0.05$ are met. Since r^* can take only integer values, the plan with smallest r^* and T^* is $r^* = 9$, $T^* = 5.25$. For this plan, $\alpha = 0.05$ and $\beta = 0.04$. If we want to keep the producer's risk at $\alpha = 0.05$, then the required plan is $r^* = 9$, $T^* = 5.45$ for which $\beta = 0.04$. Thus the two choices are:

$\alpha = 0.05$, $r^* = 9$, $T^* = 5.45$, $\beta = 0.04$, and

$\beta = 0.05$, $r^* = 9$, $T^* = 5.25$, $\alpha = 0.04$.

3.2 Example 2: Exploration of the Design Region

An important advantage of the graphical procedure is that it permits and exploration of the (T^*-r^*) plane in the region of interest. For the criteria in Example 1, plans with $\alpha = 0.05$ and $\beta \leq 0.05$ can be obtained by considering the $\alpha = 0.05$ contour as follows:

Plan No.	α	r^*	T^*	β
1	0.05	9	5.45	0.040
2	0.05	10	6.20	0.025
3	0.05	11	6.95	0.015
4	0.05	12	7.70	0.009

Plans for which $\beta = 0.05$ and $\alpha \leq 0.05$ are:

Plan No.	β	r^*	T^*	α
5	0.05	9	5.25	0.040
6	0.05	10	5.65	0.030
7	0.05	11	6.10	0.022
8	0.05	12	6.50	0.016

Some plans outside the solution region which require less testing and come close to satisfying the risks are:

Plan No.	r^*	T^*	α	β
9	8	4.70	0.05	0.062
10	8	4.85	0.06	0.05
11	7	4.00	0.05	0.09
12	7	4.40	0.075	0.05

It is easily observed that when α is kept constant, an increase in r^* implies an increase in T^* and a decrease in β resulting in an increased protection to the consumer. For constant β, the test time increases as r^* increases and α reduces which results in an added protection to the producer. In either case, the added protection is obtained at the expense of increased testing cost. Similarly, testing cost can be reduced at the expense of added risks by considering points outside the solution region. Such an exploration of the region of interest provides a vehicle for choosing a plan based on a combination of desired risks and testing costs.

3.3 Example 3: Effect of Changes in α, β and K.

The nomograms can be used to determine how sensitive the normalized test time T^* and acceptance number r^* are to changes in α, β and K. The following table is constructed by using Figures 4 and 6.

Plan No.			K	r^*	T^*
1	$\alpha = 0.05$,	$\beta \leq 0.05$	3	9	5.45
2	$\alpha = 0.025$,	$\beta \leq 0.05$	3	11	5.70
3	$\beta = 0.05$,	$\alpha \leq 0.05$	3	9	5.25
4	$\beta = 0.25$,	$\alpha \leq 0.05$	3	10	6.15
5	$\alpha = 0.05$,	$\beta \leq 0.05$	4	3	2.05

Comparing plans 1 and 2, producer's risk can be reduced to half by a 4.6% increase in T^*. Similarly, from plans 3 and 4, the consumer's risk can be reduced to half by a 17% increase in test time. An increase in the discrimination ratio leads to a considerable reduction in T^* and r^*. This is expected since less testing should be called for while distinguishing between values of θ farther apart.

The effect of change in K needs further scrutiny. Let θ_0 be 1200. Then plans 1 and 5 can be written as:

Plan			θ_o	θ_1	r^*	T^*
1(a)	$\alpha = 0.05$,	$\beta = 0.04 < 0.05$	1200	400	9	5.45
5(a)	$\alpha = 0.05$,	$\beta = 0.04 < 0.05$	1200	300	3	2.05
6	$\alpha = 0.05$,	$\beta = 0.14$	1200	400	3	2.05

Plan 6 is obtained from Figure 4 corresponding to $r^* = 3$, $T^* = 2.05$ and $\theta_1 = 400$. Plans 5 and 6 are identical since they have the same T^* and r^*. Therefore, the effect of an increase in K is the same as that of an increase in the consumer's risk keeping the producer's risk constant.

3.4 Example 4: Design with Engineering Constraints

Often engineering considerations will necessitate constraints on the allowable T^* and r^*. For example, the cost of testing or the amount of available test time may require that T^* be less than or equal to some pre-specified value. Similarly, the number of available equipments or the cost of repair may dictate limitation on r^*. In either case, the graphical procedure allows the determination of suitable sampling plans.

(a) Let us suppose that $T^* \leq 4$ and $K = 3$. Let the desired criteria be $\alpha = \beta \leq 0.05$. The restriction on T^* implies that the criteria may or may not be met in practice. If the criteria are not met, various alternatives may be presented as follows.

Referring to Figure 4, the feasible region lies to the left of $T^* = 4$ line. Since the point of intersection of $\alpha = 0.05$ and $\beta = 0.05$ contours is to the right of $T^* = 4$, the specified criteria cannot be met. We now explore the triangular region enclosed by $\alpha = 0.10$, $\beta = 0.10$ and $T^* = 4$ lines. Such an exploration ensures that none of the risks exceed 0.10. Some feasible plans are:

Plan No.	r^*	T^*	α	β
1	7	4.0	0.050	0.090
2	6	3.5	0.065	0.100
3	6	3.7	0.080	0.080
4	6	3.9	0.100	0.055
5	5	3.1	0.095	0.095

From the viewpoint of risks, plan 2 is inferior to plan 1 and plan 5 is inferior to plan 3. The choice lies between plans 1, 3 and 4.

(b) Suppose that a maximum of 5 failures are allowed. T^* is not restricted, K = 3 and the desired risks are $\alpha = \beta = 0.10$. This specification corresponds to a censored plan with $r^* \leq 5$. From Goel and Joglekar [1], the corresponding truncated plan has $r^* \leq 4$.

Referring to the nomogram for K = 3, it is observed that the desired criteria cannot be met. To obtain a suitable plan, the triangular region bounded by $\alpha = 0.15$, $\beta = 0.15$ and $r^* = 4$ is explored as follows.

Plan No.	r^*	T^*	α	β
1	4	2.40	0.10	0.15
2	4	2.55	0.12	0.11
3	4	2.70	0.15	0.07
4	3	2.00	0.15	0.15

A suitable plan can be chosen by a joint consideration of risks and test criteria. If plan 2 is selected, the corresponding censored plan is $r^* = 5$, $T^* = 2.55$.

3.5 Example 5: Design with Partial Information

In some cases, the producer may be able to specifiy a numerical value for α but the consumer may not be able to specify a numerical value for β and vice-versa. For illustration, let $\alpha \leq 0.05$ and K = 3. Then a set of alternatives may be presented as follows

Plan No.	α	r^*	T^*	β
1	0.15	2	1.35	0.23
2	0.15	3	2.05	0.14
3	0.15	4	2.80	0.08
4	0.15	5	3.60	0.045
5	0.15	6	4.35	0.025
6	0.15	7	5.20	0.015

A suitable plan can be selected by considering the limitations on testing.

4. CONCLUDING REMARKS

The nomograms given in this paper can be used to design all the six types of single sample plans described earlier by using the interrelationships developed in Goel and Joglekar [1]. For the design criteria in Example 1, the six plans are:

Truncated Plan for a System: $T^* = 5.25$, $r^* = 9$

Censored Plan for a System: $T^* = 5.25$, $r^* = 10$

Truncated Plan for a Lot With Replacement: $n = 20$, $t_0^* = 0.26$, $r^* = 9$

Censored Plan for a Lot With Replacement: $n = 20$, $T^* = 5.25$, $r^* = 10$

Truncated Plan for a Lot Without Replacement: $n = 20$, $t_0^* = 0.35$, $r^* = 9$

Censored Plan for a Lot Without Replacement: $n = 20$, $T^* = 5.25$, $r^* = 10$

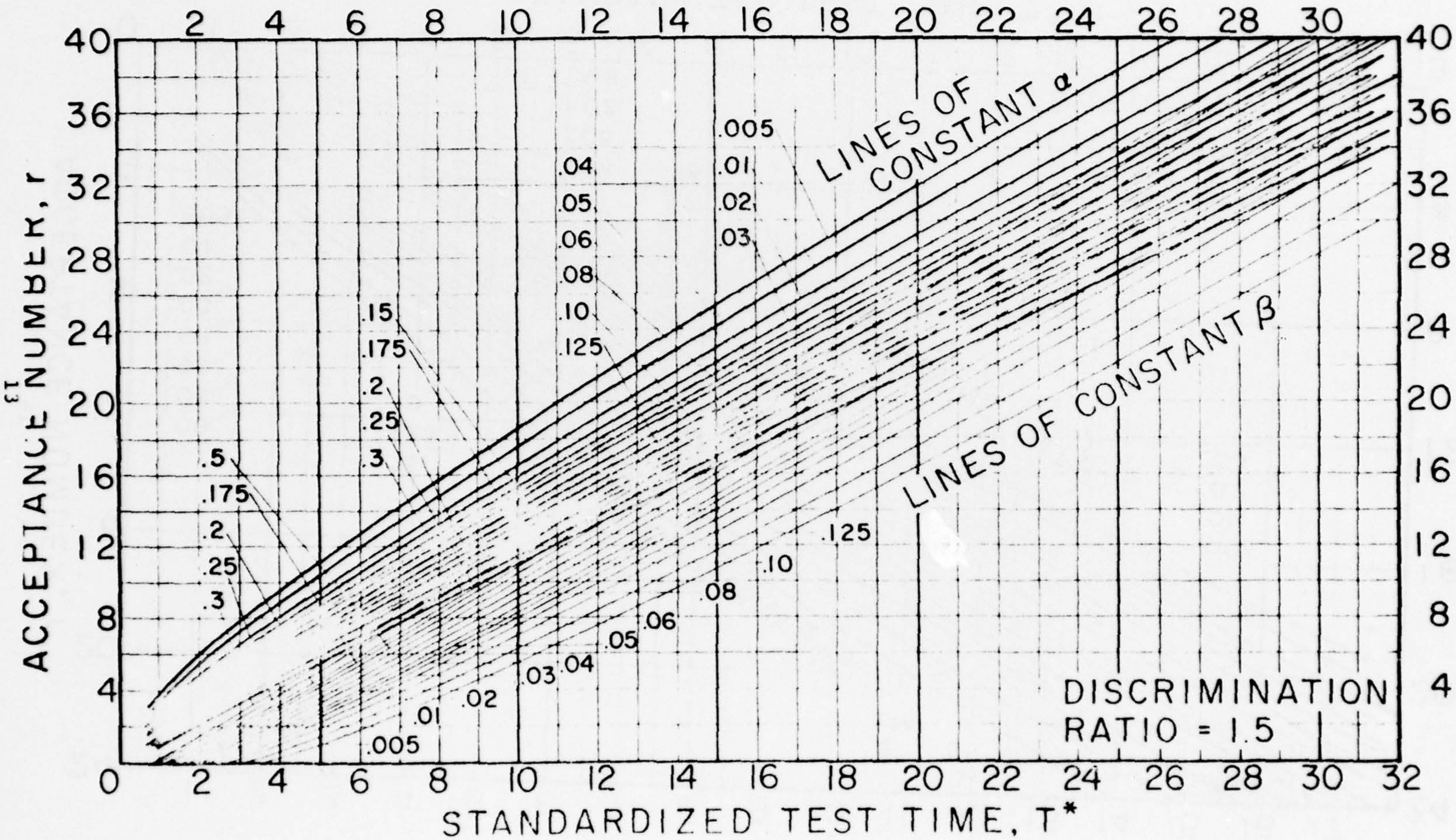
The same graphical procedure is applicable for the design of single sample plans when θ is a random variable. For a detailed description of the various risk criteria that arise in this situation as well as plan design the reader is referred to Goel and Joglekar [1].

REFERENCES

1. Goel, A.L. and Joglekar, A.M. (1976) "Reliability Acceptance Sampling Plans Based Upon Prior Distribution - Part IV: Design of Testing Plans." Technical Report, Department of Industrial Engineering and Operations Research, Syracuse University, Syracuse, New York.

2. MIL-STD-781 B (1967) Reliability Tests: Exponential Distributions, Department of Defense, November 15, 1967.

Figure 1: Nomograms for Discrimination Ratio 1.5



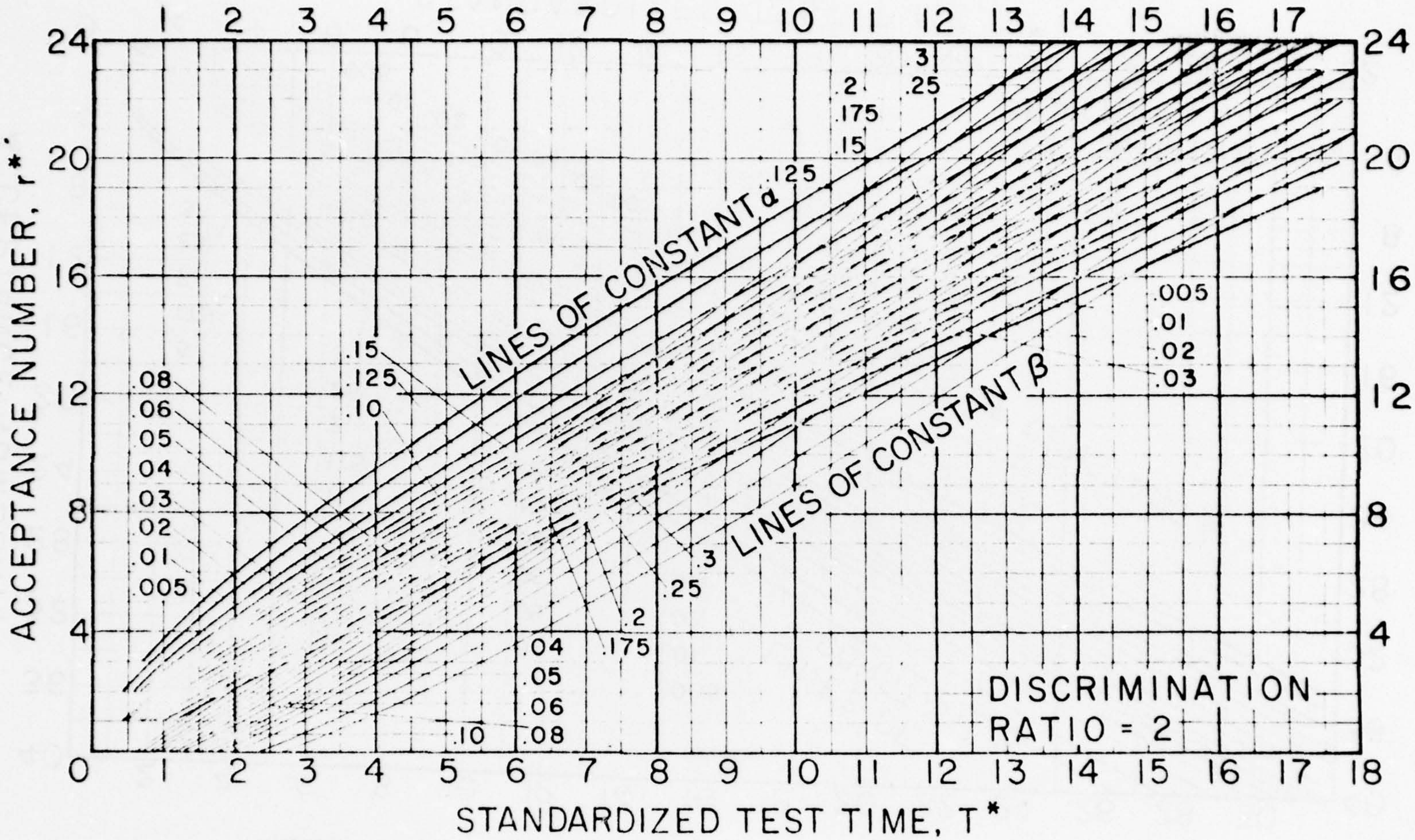


Figure 2: Nomograms for Discrimination Ratio 2

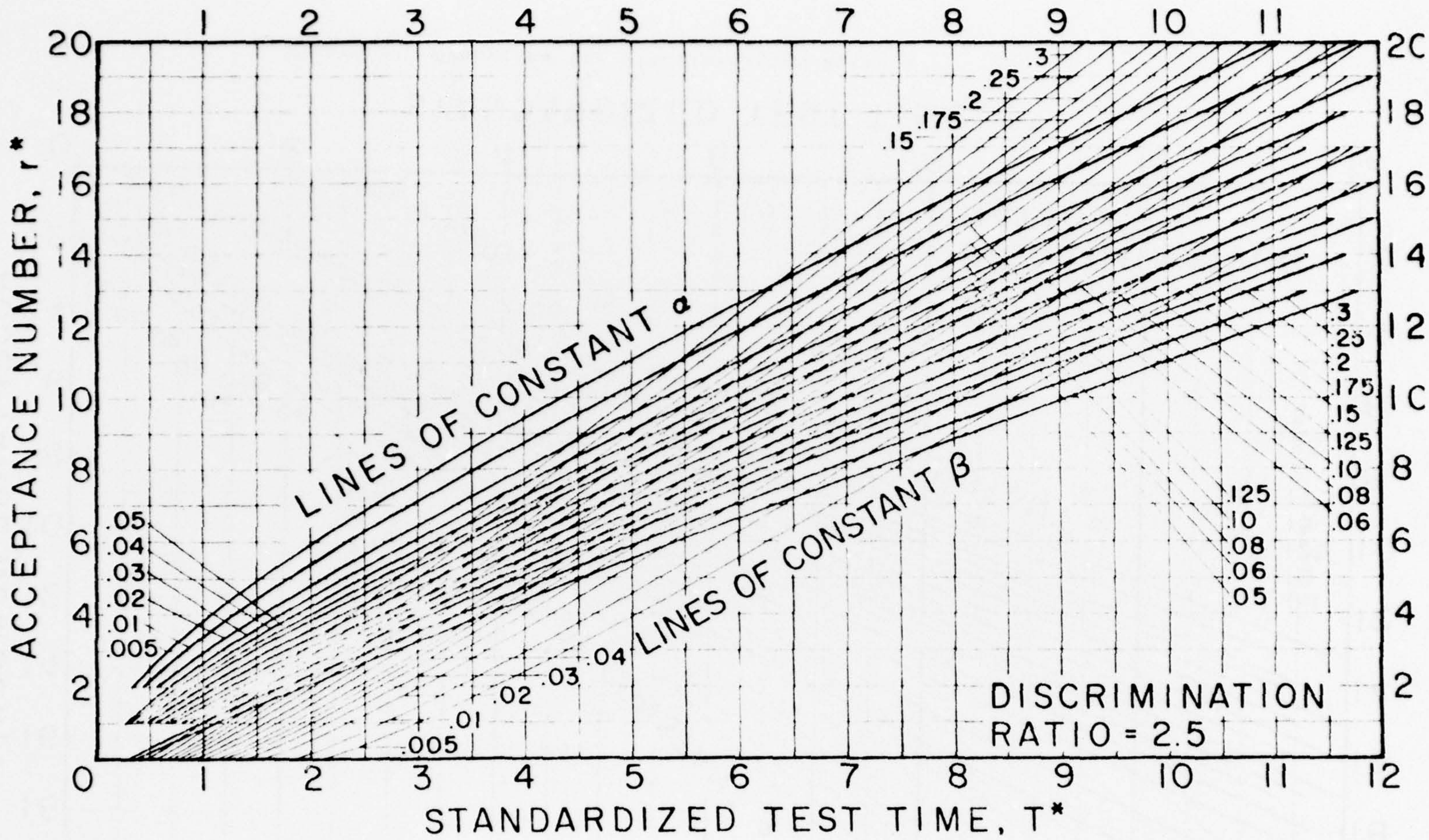
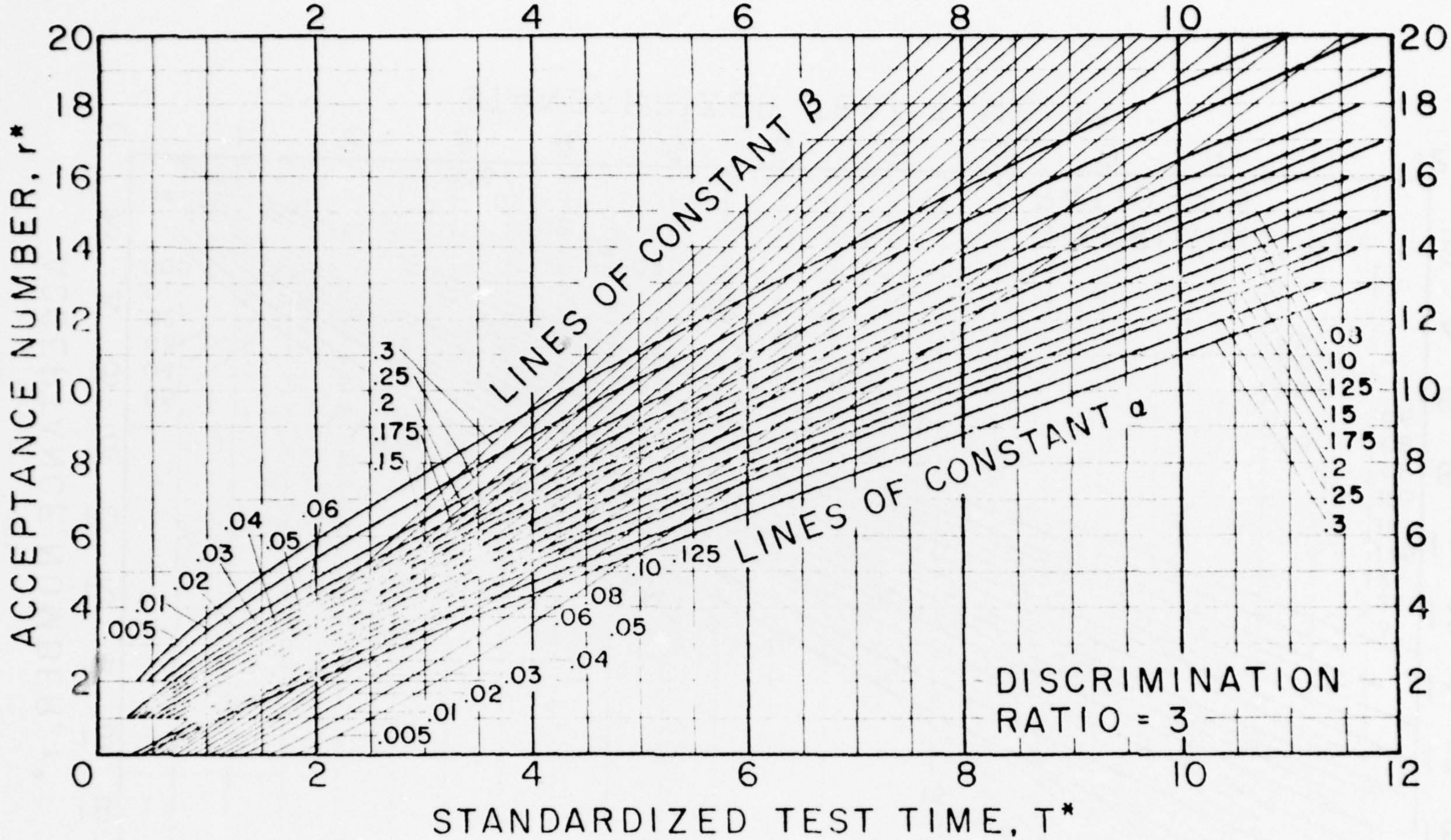


Figure 3: Nomograms for Discrimination Ratio 2.5



<u>Figure 4</u>: Monograms for Discrimination Ratio 3

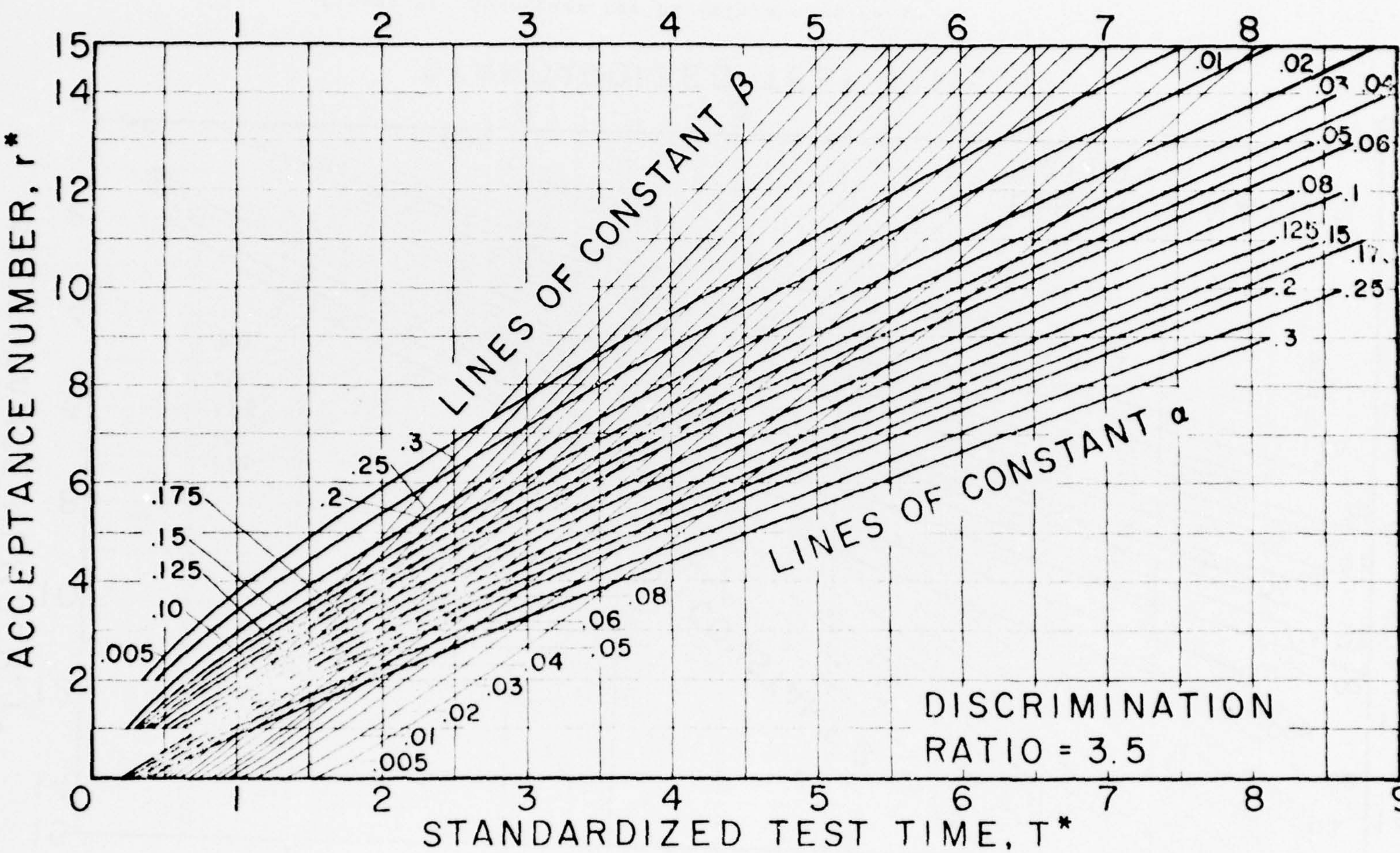


Figure 5: Nomograms for Discrimination Ratio 3.5

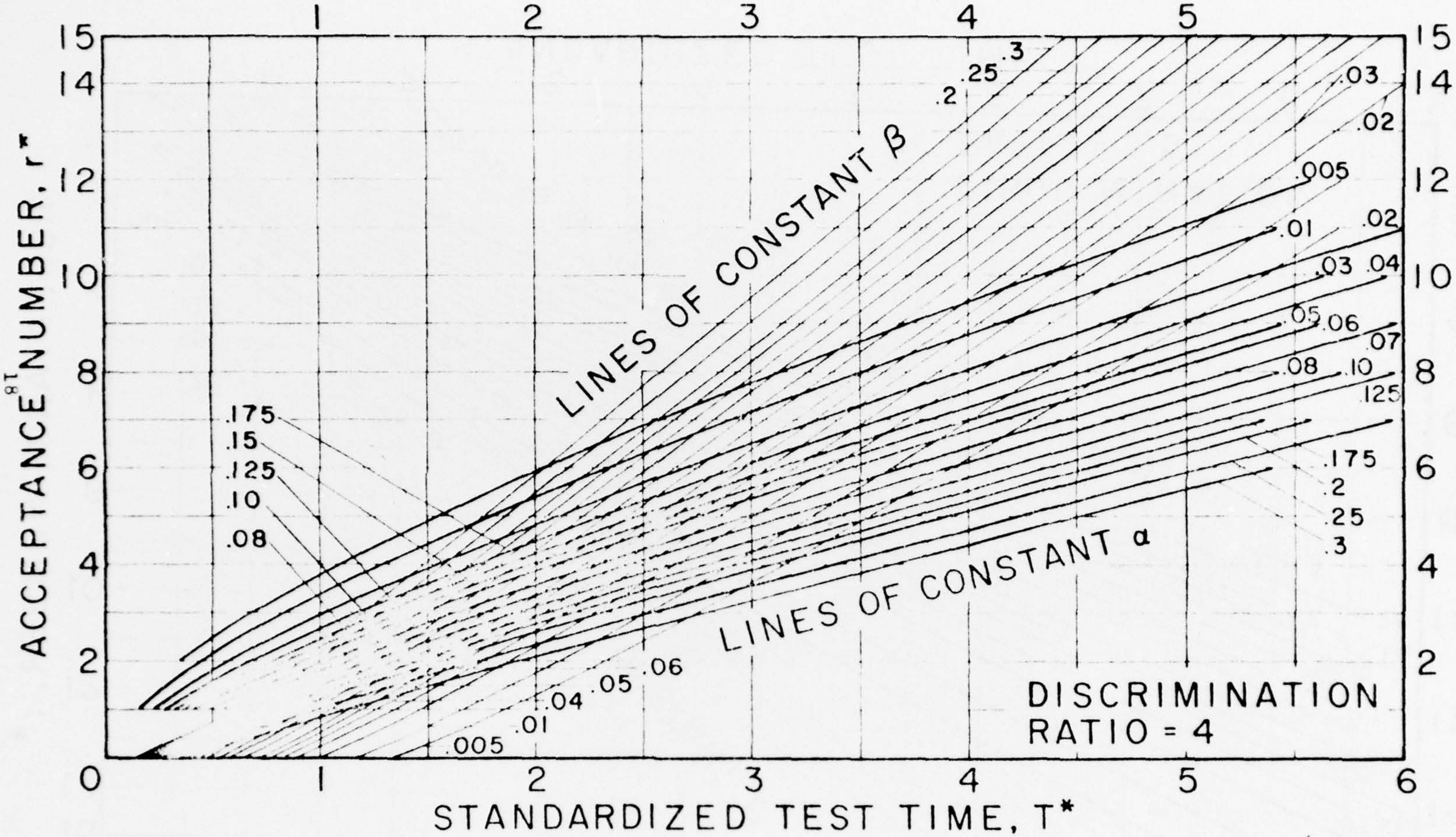


Figure 6: Nomograms for Discrimination Ratio 4

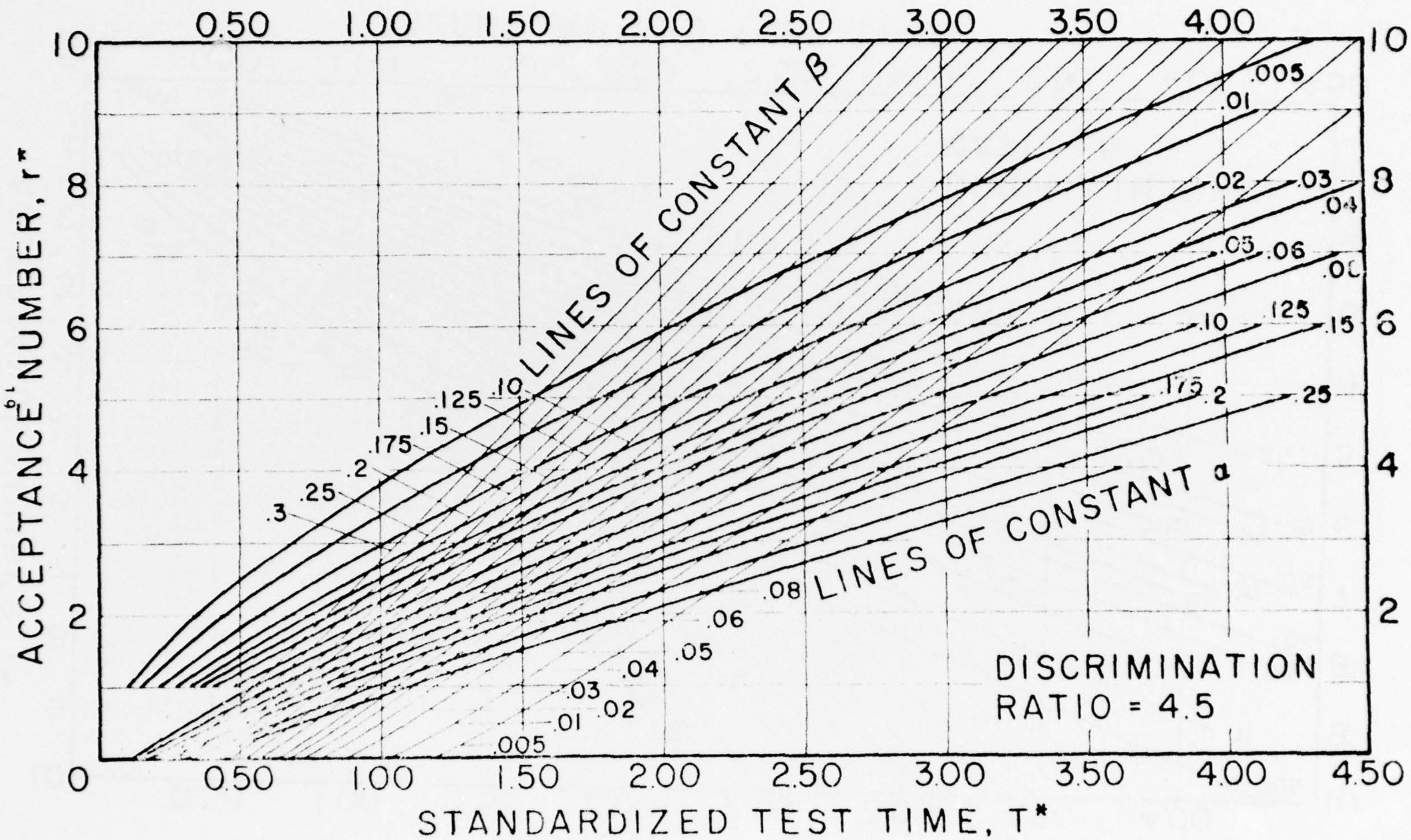


Figure 7: Nomograms for Discrimination Ratio 4.5

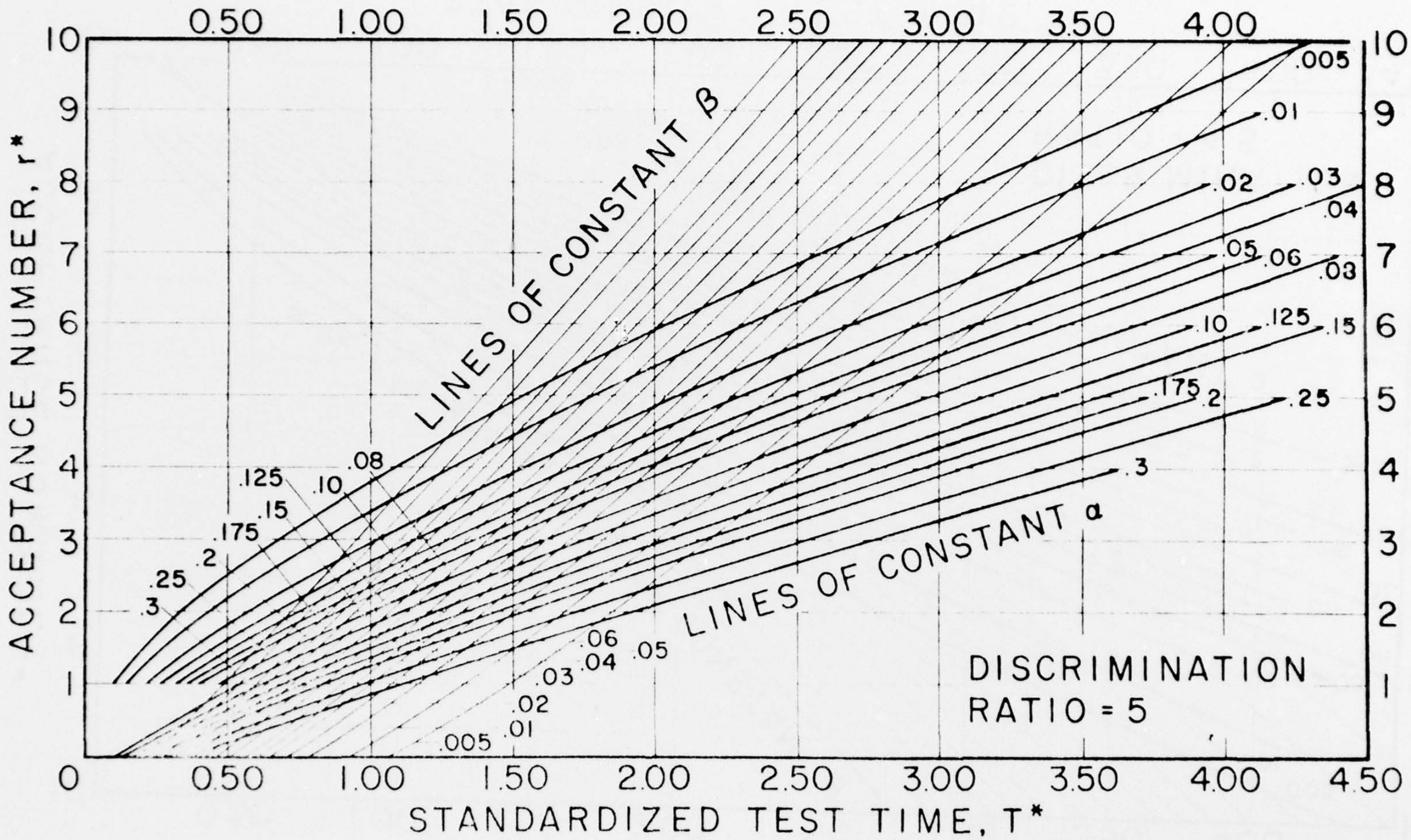


Figure 8: Nomograms for Discrimination Ratio 5

METRIC SYSTEM

BASE UNITS:

Quantity	Unit	SI Symbol	Formula
length	metre	m	...
mass	kilogram	kg	...
time	second	s	...
electric current	ampere	A	...
thermodynamic temperature	kelvin	K	...
amount of substance	mole	mol	...
luminous intensity	candela	cd	...

SUPPLEMENTARY UNITS:

Quantity	Unit	SI Symbol	Formula
plane angle	radian	rad	...
solid angle	steradian	sr	...

DERIVED UNITS:

Quantity	Unit	SI Symbol	Formula
Acceleration	metre per second squared	...	m/s
activity (of a radioactive source)	disintegration per second	...	(disintegration)/s
angular acceleration	radian per second squared	...	rad/s
angular velocity	radian per second	...	rad/s
area	square metre	...	m
density	kilogram per cubic metre	...	kg/m
electric capacitance	farad	F	A·s/V
electrical conductance	siemens	S	A/V
electric field strength	volt per metre	...	V/m
electric inductance	henry	H	V·s/A
electric potential difference	volt	V	W/A
electric resistance	ohm		V/A
electromotive force	volt	V	W/A
energy	joule	J	N·m
entropy	joule per kelvin	...	J/K
force	newton	N	kg·m/s
frequency	hertz	Hz	(cycle)/s
illuminance	lux	lx	lm/m
luminance	candela per square metre	...	cd/m
luminous flux	lumen	lm	cd·sr
magnetic field strength	ampere per metre	...	A/m
magnetic flux	weber	Wb	V·s
magnetic flux density	tesla	T	Wb/m
magnetomotive force	ampere	A	...
power	watt	W	J/s
pressure	pascal	Pa	N/m
quantity of electricity	coulomb	C	A·s
quantity of heat	joule	J	N·m
radiant intensity	watt per steradian	...	W/sr
specific heat	joule per kilogram-kelvin	...	J/kg·K
stress	pascal	Pa	N/m
thermal conductivity	watt per metre-kelvin	...	W/m·K
velocity	metre per second	...	m/s
viscosity, dynamic	pascal-second	...	Pa·s
viscosity, kinematic	square metre per second	...	m/s
voltage	volt	V	W/A
volume	cubic metre	...	m
wavenumber	reciprocal metre	...	(wave)/m
work	joule	J	N·m

SI PREFIXES:

Multiplication Factors	Prefix	SI Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto*	h
10 = 10^{1}	deka*	da
0.1 = 10^{-1}	deci*	d
0.01 = 10^{-2}	centi*	c
0.001 = 10^{-3}	milli	m
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